Solutions to the exercises, specified in the example of the ExSol package

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Exercise 2-1: Solve the following equation for \( x \in C \), with \( C \) the set of complex numbers:

\[
5x^2 - 3x = 5
\]

Solution: Let’s start by rearranging the equation, a bit:

\[
5.7x^2 - 3.1x = 5.3
\]
\[
5.7x^2 - 3.1x - 5.3 = 0
\]

The equation is now in the standard form:

\[
ax^2 + bx + c = 0
\]

For quadratic equations in the standard form, we know that two solutions exist:

\[
x_{1,2} = \frac{-b \pm \sqrt{d}}{2a}
\]

with

\[
d = b^2 - 4ac
\]

If we apply this to our case, we obtain:

\[
d = (-3.1)^2 - 4 \cdot 5.7 \cdot (-5.3) = 130.45
\]

and

\[
x_1 = \frac{3.1 + \sqrt{130.45}}{11.4} = 1.27
\]
\[
x_2 = \frac{3.1 - \sqrt{130.45}}{11.4} = -0.73
\]

The proposed values \( x = x_1, x_2 \) are solutions to the given equation.

Exercise 2-2: Consider a 2-dimensional vector space equipped with a Euclidean distance function. Given a right-angled triangle, with the sides \( A \) and \( B \) adjacent to the right angle having lengths, 3 and 4, calculate the length of the hypotenuse, labeled \( C \).

Solution: This calls for application of Pythagoras’ theorem, which tells us:

\[
\|A\|^2 + \|B\|^2 = \|C\|^2
\]

and therefore:

\[
\|C\| = \sqrt{\|A\|^2 + \|B\|^2}
\]
\[
= \sqrt{3^2 + 4^2}
\]
\[
= \sqrt{25} = 5
\]

Therefore, the length of the hypotenuse equals 5.